

# The Belief Functions Theory for Sensors Localization in Indoor Wireless Networks

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**Abstract.** This paper investigates the usage of the belief functions theory to localize sensors in indoor environments. The problem is tackled as a zoning localization where the objective is to determine the zone where the mobile sensor resides at any instant. The proposed approach uses the belief functions theory to define an evidence framework, for estimating the most probable sensor's zone. Real experiments demonstrate the effectiveness of this approach as compared to other localization methods.

## 1 Introduction

Localization is an essential issue in wireless sensor networks to process the information retrieved by sensor nodes. This paper proposes a zoning-based localization technique that makes use of the belief functions theory (BFT) to combine evidence revealed at each sensor. The proposed approach is constituted of two phases. In an offline phase, received signal strength indicators (RSSIs) received from neighboring WiFi Access Points (APs) are collected in each zone and a fingerprints database is built. The kernel density estimation is then used to represent the measurements and set mass functions over the zones. In the same manner, mass functions are also constructed over supersets of zones, by concatenating zones data. In an online phase, the collected RSSIs of a mobile sensor are used in the belief functions framework to determine its zone. Since APs are not completely reliable, their associated masses are discounted according to their error rate. Afterwards, the fusion of all evidence is carried by combining masses using the conjunctive rule of combination. Finally, the pignistic transformation is applied to assign evidence to singleton sets that are the original zones. The zone having the highest evidence is then selected. Experiments on real data illustrate the performance of the belief functions framework for localization of sensors against other localization techniques.

## 2 Belief functions localization method

### 2.1 Problem formulation

The localization problem is tackled in the following manner. Let  $N_Z$  be the number of zones of the targeted area, denoted by  $Z_j$ ,  $j \in \{1, 2, \dots, N_Z\}$ . Let

$N_{AP}$  be the number of detected APs, denoted by  $AP_k$ ,  $k \in \{1, 2, \dots, N_{AP}\}$ . Let  $\rho_{j,k,r}$ ,  $r \in \{1, \dots, \ell_j\}$ , be the set of  $\ell_j$  measurements collected in an offline phase in the zone  $Z_j$  with respect to  $AP_k$ . Let  $\boldsymbol{\rho}_t$  be the vector of  $N_{AP}$  RSSI measurements collected by the mobile sensor at the instant  $t$  from all the APs. The aim of the proposed algorithm is to determine the zone  $\hat{Z}_{j,t}$  having the highest evidence,  $\hat{Z}_{j,t} = \arg \max_{Z_j} \mathcal{W}_t(Z_j)$ , such that  $\mathcal{W}_t(Z_j)$  represents the evidence in having the mobile sensor of observation  $\boldsymbol{\rho}_t$  residing in the zone  $Z_j$  at instant  $t$ .

## 2.2 Mass assignment

In the offline phase, the kernel density estimation (KDE) is proposed to model the distribution of the collected measurements  $\rho_{j,k,r}$ ,  $r \in \{1, \dots, \ell_j\}$ , of each zone  $j$  according to each AP  $AP_k$ . The density estimate  $Q_{KDE,Z_j,k}(\cdot)$  is calculated as,

$$Q_{KDE,Z_j,k}(\cdot) = \frac{1}{\ell_j \times h} \sum_{r=1}^{\ell_j} \mathcal{K}\left(\frac{\cdot - \rho_{j,k,r}}{h}\right), \quad (1)$$

where  $\mathcal{K}(u)$  is a Gaussian kernel, and  $h$  its bandwidth,

$$\mathcal{K}(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}. \quad (2)$$

A practical approach to determine  $h$  is to maximize the pseudo-likelihood leave-one-out cross validation,

$$ML(h) = \ell_j^{-1} \sum_{r=1}^{\ell_j} \log \left[ \sum_{r' \neq r} \mathcal{K}\left(\frac{\rho_{j,k,r'} - \rho_{j,k,r}}{h}\right) \right] - \log[(\ell_j - 1)h]. \quad (3)$$

The computations are conducted in the same manner for all the supersets of the zones. Let  $A$  be a superset of zones. Then, the RSSIs related to all zones of  $A$  are considered to construct the kernel density estimate related to  $A$ , denoted  $Q_{KDE,A,k}(\cdot)$  as in Eq. (1). In the online phase, once a new measurement  $\boldsymbol{\rho}_t = (\rho_{t,1}, \dots, \rho_{t,N_{AP}})$  is carried for localization, the kernel density estimates obtained in the offline phase is used with the belief functions theory to determine the zone of the sensor. Let  $\mathcal{Z}$  be the set of all possible zones  $Z_j$ ,  $j \in \{1, \dots, N_Z\}$ , and let  $2^{\mathcal{Z}}$  be the set of all supersets of  $\mathcal{Z}$ , i.e,  $2^{\mathcal{Z}} = \{\{Z_1\}, \dots, \mathcal{Z}\}$ . The mass function (MF)  $m_{AP_k,t} : 2^{\mathcal{Z}} \rightarrow [0, 1]$ , defined according to  $AP_k$  is calculated as follows [1],

$$m_{AP_k,t}(A) = Q_{KDE,A,k}(\boldsymbol{\rho}_{t,k}). \quad (4)$$

## 2.3 Discounting operation

The detected APs are not completely reliable. Indeed, each AP could yield an erroneous interpretation of evidence for some observations. In order to correct this, one can discount the MFs of Eq. (4) by taking into account the error rate

of the AP. The discounted MF  $\alpha m_{AP_k,t}$  related to  $AP_k$  having an error rate  $\alpha_k$  is deduced from  $m_{AP_k,t}$  as follows [2],

$$\alpha m_{AP_k,t}(A) = \begin{cases} (1 - \alpha_k)m_{AP_k,t}(A), & \text{if } A \in 2^{\mathcal{Z}}, A \neq \mathcal{Z}; \\ \alpha_k + (1 - \alpha_k)m_{AP_k,t}(A), & \text{if } A = \mathcal{Z}. \end{cases} \quad (5)$$

By doing this, the amounts of evidence given to the subsets of  $\mathcal{Z}$  are reduced, and the remaining evidence is given to the whole set  $\mathcal{Z}$ . The source  $AP_k$  is assumed not reliable if, according to an observation  $\rho_{k,\cdot}$  being truly in  $A$ , it associates more evidence to any set other than  $A$ , that is, the mass associated to  $A$  is less than the mass of another subset of  $2^{\mathcal{Z}}$ . Let  $\epsilon_k(A)$  be the error rate related to the set  $A$  with respect to  $AP_k$ . Then,

$$\epsilon_k(A) = \int_{\mathbb{D}_{k,A}} Q_{KDE,A,k}(\rho) d\rho, \quad (6)$$

such that  $\mathbb{D}_{k,A}$  is the domain of error of set  $A$  according to  $AP_k$ , defined as,

$$\mathbb{D}_{k,A} = \{\rho \mid Q_{KDE,A,k}(\rho) \leq \max_{A' \in 2^{\mathcal{Z}}, A' \neq A} (Q_{KDE,A',k}(\rho))\}. \quad (7)$$

The error rate  $\alpha_k$  of  $AP_k$  is then the average error of all subsets according to this AP, namely

$$\alpha_k = \frac{\sum_{A \in 2^{\mathcal{Z}}} \epsilon_k(A)}{2^{|\mathcal{Z}|} - 1}. \quad (8)$$

## 2.4 Evidence fusion

The evidence is then combined by aggregating the information coming from all the detected APs [3]. The mass functions can then be combined using the conjunctive rule of combination as follows,

$$m_{\cap,t}(A) = \sum_{\substack{A^{(k)} \in 2^{\mathcal{Z}} \\ \cap_k A^{(k)} = A}} \prod_{k=1}^{N_{AP}} \alpha m_{AP_k,t}(A^{(k)}), \quad (9)$$

$\forall A \in 2^{\mathcal{Z}}$ , with  $A^{(k)}$  is the subset  $A$  with respect to the Access Point  $AP_k$ .

## 2.5 Decision

An adequate notion of the BFT to attribute masses to singleton sets  $A \in 2^{\mathcal{Z}}$  is the pignistic level [4]. It is defined as follows,

$$BetP_t(A) = \sum_{A' \subseteq A} \frac{m_{\cap,t}(A')}{|A'|}, \quad (10)$$

The zone  $\hat{Z}_{j,t}$  having the highest evidence at instant  $t$  is then selected,

$$\hat{Z}_{j,t} = \arg \max_{Z_j} BetP_t(\{Z_j\}), j \in \{1, \dots, N_Z\}. \quad (11)$$

### 3 Experiments

Real experiments are conducted in the Living Lab of the University of Technology of Troyes, France. The considered floor of approximated area of  $500\text{ m}^2$  is partitioned into 19 zones, where 12 AP networks could be detected. A Set of 50 measurements is taken in each zone, of which 30 are randomly used to construct the databases, and the others are kept for test. The proposed approach is compared to other localization techniques such as weighted  $k$ -nearest neighbors algorithm (WKNN) presented in [5] and a Multinomial logistic regression (MLR) presented in [6]. The proposed method achieves an accuracy of 85.26% outperforming the WKNN with 83.82% and the MLR with 82.94%.

### 4 Conclusion and future work

This paper presented a belief functions framework for localization of sensors in indoor wireless networks. The kernel density estimation was used to set mass functions, and the belief functions theory combined evidence to determine the sensor's zone. Experiments on real data prove the effectiveness of the approach as compared to other localization techniques. Future work will focus on using the mobility as another source of information.

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